MATEMÁTICAS

2º BACHILLERATO
Integrales

$$\int \sqrt{x^3}\,dx = \int x^{3/2}\,dx = \frac{x^{5/2}}{5/2} = \frac{2}{5}\cdot x^2\sqrt{x} + C$$

$$\int (2x^2-5x+3)\,dx = \frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x + C$$

$$\int (1-x)\sqrt{x}\,dx = \int (x^{1/2} - x^{3/2})\,dx = \frac{2}{3}x\sqrt{x} - \frac{2}{5}x^2\sqrt{x} + C$$

$$\int (3x+4)^2\,dx = \frac{1}{3}\int 3(3x+4)^2\,dx = \frac{1}{9}(3x+4)^3 + C$$

$$\int \frac{x^3+5x^2-4}{x^2}\,dx = \int \left(x + 5 - \frac{4}{x^2}\right)dx = \frac{x^2}{2} + 5x + \frac{4}{x} + C$$

$$\int (x^3+2)^2x^2\,dx = \frac{1}{3}\int 3x^2(x^3+2)^2\,dx = \frac{1}{9}(x^3+2)^3 + C$$

$$\int 7x^2\sqrt{x^3+2}\,dx = \frac{7}{3}\int 3x^2(x^3+2)^{1/2}\,dx = \frac{14}{9}(x^3+2)\sqrt{x^3+2} + C$$

$$\int \frac{5x^2}{(8x^3+5)^3}\,dx = \int 5x^2(8x^3+5)^{-3}\,dx = \frac{5}{24}\int 24x^2(8x^3+5)^{-3}\,dx = \frac{-5}{48(8x^3+5)^2} + C$$

$$\int \frac{x^3}{\sqrt[4]{x^4+3}}\,dx = \frac{1}{4}\int 4x^3(x^4+3)^{-1/4}\,dx = \frac{1}{3}(x^4+3)^{3/4} + C = \frac{1}{3}\sqrt[4]{(x^4+3)^3} + C$$

$$\int 3x\sqrt{1-2x^2}\,dx = -\frac{3}{4}\int -4x^2(1-2x^2)^{1/2}\,dx = -\frac{1}{2}(1-2x^2)\sqrt{1-2x^2} + C$$

$$\int \frac{x+3}{\sqrt[3]{x^2+6x}}\,dx = \int (x+3)(x^2+6x)^{-1/3}\,dx = \frac{1}{2}\int 2(x+3)(x^2+6x)^{-1/3}\,dx = \frac{3}{4}\sqrt[3]{(x^2+6x)^2} + C$$

$$\int \sqrt[4]{1-x^2}\cdot x\,dx = -\frac{1}{2}\int -2x\sqrt[4]{1-x^2}\,dx = -\frac{2}{5}(1-x^2)\sqrt[4]{1-x^2} + C$$

$$\int \sqrt{x^2+5x^4}\,dx = \int \sqrt{x^2(1+5x^2)}\,dx = \int x\sqrt{1+5x^2}\,dx = \int x(1+5x^2)^{1/2}\,dx = \frac{1}{15}(1+5x^2)\sqrt{1+5x^2} + C$$

$$\int \frac{(1+x)^2}{\sqrt{x}}\,dx = \int \frac{1+x^2+2x}{\sqrt{x}}\,dx = \int (x^{-1/2} + x^{3/2} + 2x^{1/2})\,dx = 2\sqrt{x} + \frac{2}{5}x^2\sqrt{x} + \frac{4}{3}x\sqrt{x} + C$$

$$\int \frac{x^2+2x}{(x+1)^2}\,dx = \left[\begin{array}{l|l} x^2+2x & x^2+2x+1 \\ \hline -x^2-2x-1 & 1 \\ -1 & \end{array}\right]\left[\frac{D}{d} = c + \frac{r}{d}\right] = \int \left(1 - \frac{1}{(x+1)^2}\right)dx = x + \frac{1}{x+1} + C$$

$$\int \frac{dx}{2x-3} = \frac{1}{2}L|2x-3| + C$$

$$\int \frac{x}{x^2-1}\,dx = \frac{1}{2}L|x^2-1| + C$$

$$\int \frac{x^2}{1-2x^3}dx = -\frac{1}{6} L|1-2x^3| + C$$

$$\int \frac{x+2}{x+1}dx = \left[\text{grado num.} \geq \text{grado denom} \Rightarrow \text{Dividir polim.}\right] = \int \left(1 + \frac{1}{x+1}\right)dx = x + L|x+1| + C$$

$$\int e^{-x}dx = -\int -e^{-x}dx = -e^{-x} + C$$

$$\int a^{5x}dx = \frac{1}{5\,La}\int a^{5x}\cdot 5\cdot La\,dx = \frac{a^{5x}}{5\,La} + C$$

$$\int e^{3x}dx = \frac{1}{3}e^{3x} + C$$

$$\int \frac{e^{1/x}}{x^2} = \int \frac{1}{x^2}\cdot e^{1/x}dx = -e^{1/x} + C$$

$$\int (e^x+1)^4\cdot e^x\,dx = \frac{1}{5}(e^x+1)^5 + C$$

$$\int \frac{dx}{e^x+1} = \int \frac{e^{-x}}{e^{-x}(e^x+1)}dx = \int \frac{e^{-x}}{1+e^{-x}}dx = -L(1+e^{-x}) + C$$

$$\int \operatorname{sen}\frac{x}{2}dx = -2\int -\frac{1}{2}\operatorname{sen}\frac{x}{2}dx = -2\cos\frac{x}{2} + C$$

$$\int \cos 3x\,dx = \frac{1}{3}\operatorname{sen} 3x + C$$

$$\int x\cdot \operatorname{tg} x^2 dx = -\frac{1}{2}\int \frac{-2x\cdot \operatorname{sen} x^2}{\cos x^2}dx = -\frac{1}{2}L|\cos x^2| + C$$

$$\int \frac{L^3 x}{x}dx = \int \frac{1}{x}\cdot L^3 x\,dx = \frac{L^4 x}{4} + C$$

$$\int \frac{\sec\sqrt{x}}{\sqrt{x}}dx = \int \frac{1}{\sqrt{x}}\cdot\frac{\sec\sqrt{x}\,(\sec\sqrt{x}+\operatorname{tg}\sqrt{x})}{\sec\sqrt{x}+\operatorname{tg}\sqrt{x}}dx = 2\int \frac{1}{2\sqrt{x}}\cdot\frac{\sec^2\sqrt{x}+\sec\sqrt{x}\,\operatorname{tg}\sqrt{x}}{\sec\sqrt{x}+\operatorname{tg}\sqrt{x}}dx =$$

$$= 2\cdot L|\sec\sqrt{x}+\operatorname{tg}\sqrt{x}| + C$$

<u>otra forma</u> (mucho más corta):

$$\int \frac{\sec\sqrt{x}}{\sqrt{x}}dx = \int \frac{\frac{dx}{\sqrt{x}}}{\cos\sqrt{x}} = \int \frac{\frac{dx}{\sqrt{x}}}{\operatorname{sen}\left(\frac{\pi}{2}-\sqrt{x}\right)} = \int \frac{\frac{dx}{\sqrt{x}}}{\operatorname{sen} 2\cdot\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)} = \int \frac{\frac{dx}{\sqrt{x}}}{2\operatorname{sen}\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)} =$$

dividir num. y denom. entre $\cos^2\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)$

$$= \int \frac{\frac{dx}{\sqrt{x}\cdot\cos^2(\pi/4-\sqrt{x}/2)}}{2\operatorname{tg}\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)} = \int \frac{\frac{1}{\sqrt{x}}\cdot\sec^2\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)}{2\operatorname{tg}\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)}dx = 2L\left|\operatorname{tg}\left(\frac{\pi}{4}-\frac{\sqrt{x}}{2}\right)\right| + C$$

$$\int \frac{dx}{\sqrt{28-12x-x^2}} = \left[28-12x-x^2 = b^2-(x-a)^2;\ a=-6,\ b^2=64\right] = \int \frac{dx}{\sqrt{64-(x+6)^2}} =$$

$$= \int \frac{\frac{1}{8}\,dx}{\sqrt{1-\left(\frac{x+6}{8}\right)^2}} = \operatorname{arsen}\left(\frac{x+6}{8}\right)+C$$

$$\int \frac{x+3}{\sqrt{5-4x-x^2}}\,dx = \int \frac{x}{\sqrt{5-4x-x^2}}\,dx + \int \frac{3\,dx}{\sqrt{5-4x-x^2}} = \int \frac{x+2-2}{\sqrt{5-4x-x^2}}\,dx + \int \frac{3\,dx}{\sqrt{5-4x-x^2}} =$$

$$= \int \frac{x+2}{\sqrt{5-4x-x^2}}\,dx + \int \frac{1\,dx}{\sqrt{5-4x-x^2}} = -\frac{1}{2}\int (-2x-4)(5-4x-x^2)^{-1/2}\,dx + \int \frac{dx}{\sqrt{5-4x-x^2}} =$$

$$\left[-x^2-4x+5 = b^2-(x-a)^2 \Rightarrow a=-2;\ b^2=9\right]$$

$$= -\sqrt{5-4x-x^2} + \int \frac{dx}{\sqrt{9-(x+2)^2}} = -\sqrt{5-4x-x^2} + \operatorname{arsen}\left(\frac{x+2}{3}\right)+C$$

$$\int \frac{2x+3}{9x^2-12x+8}\,dx = \frac{1}{9}\int \frac{18x+27-39+39}{9x^2-12x+8}\,dx = \frac{1}{9}\int \frac{18x-12+39}{9x^2-12x+8}\,dx =$$

$$= \frac{1}{9}\int \frac{18x-12}{9x^2-12x+8}\,dx + \frac{1}{9}\int \frac{39}{9x^2-12x+8}\,dx = \left[9x^2-12x+8 = (3x-2)^2+4\right] =$$

$$= \frac{1}{9}L\left|9x^2-12x+8\right| + \frac{13}{18}\operatorname{artg}\left(\frac{3x-2}{2}\right)+C$$

$$\int \frac{x+2}{\sqrt{4x-x^2}}\,dx = -\int \frac{-2x-4+8-8}{2\sqrt{4x-x^2}}\,dx = -\int \frac{-2x+4}{2\sqrt{4x-x^2}}\,dx + 8\int \frac{dx}{2\sqrt{4x-x^2}} =$$

$$= -\sqrt{4x-x^2} + 8\int \frac{dx}{2\sqrt{4x-x^2}} \quad \left[-x^2+4x = b^2-(x-a)^2 \Rightarrow b^2=4;\ a=2\right]$$

$$= -\sqrt{4x-x^2} + \int \frac{4\,dx}{\sqrt{4-(x-2)^2}} = -\sqrt{4x-x^2} + 4\operatorname{arsen}\left(\frac{x-2}{2}\right)+C$$

$$\int \frac{5-4x}{\sqrt{-4x^2+12x-8}}\,dx = \frac{1}{2}\int \frac{2(5-4x)\,dx}{\sqrt{-4x^2+12x-8}} = \frac{1}{2}\int \frac{10-8x+2-2}{\sqrt{-4x^2+12x-8}}\,dx =$$

$$= \frac{1}{2}\int \frac{-8x+12}{\sqrt{-4x^2+12x-8}}\,dx - \int \frac{dx}{\sqrt{-4x^2+12x-8}} = \sqrt{-4x^2+12x-8} - \int \frac{dx}{\sqrt{1-(2x-3)^2}} = (*)$$

$$-4x^2+12x-8 = 4(-x^2+3x-2) = 4\left(b^2-(x-a)^2\right) \Rightarrow a=3/2;\ b^2=1/4 \Rightarrow$$

$$\Rightarrow -4x^2+12x-8 = 4\left[\frac{1}{4}-(x-3/2)^2\right] = 1-4(x-3/2)^2 = 1-4\cdot\left(\frac{2x-3}{2}\right)^2 =$$

$$= 1-\cancel{4}\cdot\frac{(2x-3)^2}{\cancel{4}} = 1-(2x-3)^2$$

$$(*) = \sqrt{-4x^2+12x-8} - \frac{1}{2}\operatorname{arsen}(2x-3)+C$$

$$\int x\, Lx\, dx \quad \left[Lx = u;\ du = \frac{dx}{x};\ dv = x\,dx;\ v = \frac{x^2}{2} \right]$$

$$= \frac{x^2}{2} Lx - \int \frac{x^2}{2} \cdot \frac{dx}{x} = \frac{x^2}{2} Lx - \frac{x^2}{4} + c$$

$$\int x \cdot \operatorname{sen} x\, dx \quad [x = u;\ du = dx;\ \operatorname{sen} x\, dx = dv;\ v = -\cos x]$$

$$= -x\cos x + \int \cos x\, dx = -x \cos x + \operatorname{sen} x + K$$

$$\int x \cdot \cos 3x\, dx = \quad \left[x = u;\ du = dx;\ \cos 3x\, dx = dv;\ v = \frac{1}{3} \operatorname{sen} 3x \right]$$

$$= \frac{x}{3} \operatorname{sen} 3x - \int \frac{1}{3} \operatorname{sen} 3x\, dx = \frac{x}{3} \operatorname{sen} 3x + \frac{1}{9} \cos 3x + C$$

$$\int \frac{x}{e^x} dx = \int x \cdot e^{-x} dx \quad [x = u;\ du = dx;\ e^{-x} dx = dv;\ v = -e^{-x}]$$

$$= -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + C = -e^{-x}(1 + x) + C$$

$$\int x \cdot 2^{-x} dx \quad \left[x = u;\ du = dx;\ 2^{-x} dx = dv;\ v = \frac{-2^{-x}}{L2} \right]$$

$$= \frac{-x 2^{-x}}{L2} - \int \frac{-2^{-x}}{L2} dx = \frac{-x 2^{-x}}{L2} - \frac{2^{-x}}{L^2 2} + c$$

$$\int x^2 \cdot e^{3x} dx \quad \left[x^2 = u;\ du = 2x\,dx;\ e^{3x} dx = dv;\ v = \frac{1}{3} e^{3x} \right]$$

$$= \frac{x^2}{3} e^{3x} - \int \frac{1}{3} e^{3x} \cdot 2x\,dx = \quad \left[x = u;\ du = dx;\ e^{3x} dx = dv;\ v = \frac{1}{3} e^{3x} \right]$$

$$= \frac{x^2}{3} e^{3x} - \frac{2}{3} \left[\frac{x}{3} e^{3x} - \int \frac{1}{3} e^{3x} dx \right] = \frac{x^2}{3} e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + C$$

$$\int (x^2 - 2x + 5) e^{-x} dx = \quad [(x^2 - 2x + 5) = u;\ du = 2x - 2;\ e^{-x} dx = dv;\ v = -e^{-x}]$$

$$= -(x^2 - 2x + 5) e^{-x} + \int (2x - 2) e^{-x} dx = \quad [2x - 2 = u;\ du = 2dx;\ e^{-x} dx = dv;\ v = -e^{-x}]$$

$$= -(x^2 - 2x + 5) e^{-x} + \left[-(2x - 2) e^{-x} + \int e^{-x} \cdot 2dx \right] = -e^{-x}(x^2 + 5) + C$$

$$\int x^3 \cdot e^{-x/3} dx \quad [x^3 = u;\ du = 3x^2;\ e^{-x/3} dx = dv;\ v = -3e^{-x/3}]$$

$$= -3x^3 e^{-x/3} + \int 9x^2 e^{-x/3} dx \quad [u = x^2;\ du = 2x\,dx;\ dv = e^{-x/3} dx;\ v = -3e^{-x/3}]$$

$$= -3x^3 e^{-x/3} + 9\left[-3x^2 e^{-x/3} + \int 3e^{-x/3} \cdot 2x\,dx \right] \quad [x = u;\ du = dx;\ dv = e^{-x/3} dx;\ v = -3e$$

$$= -3x^3 e^{-x/3} - 27x^2 e^{-x/3} + 54\left[-3x e^{-x/3} + \int 3e^{-x/3} dx \right] =$$

$$= -3x^3 e^{-x/3} - 27x^2 e^{-x/3} - 162x e^{-x/3} - 486 e^{-x/3} + C$$