MATEMÁTICAS

2º BACHILLERATO
Tema 4: Estudio de la función

1. $\boxed{y = \dfrac{x^2}{x^2-4x+3}}$

$D = \mathbb{R} - \{1,3\}$

Cortes: $(0,0)$

Signo: $+(-\infty,1) \cup (3,+\infty)$

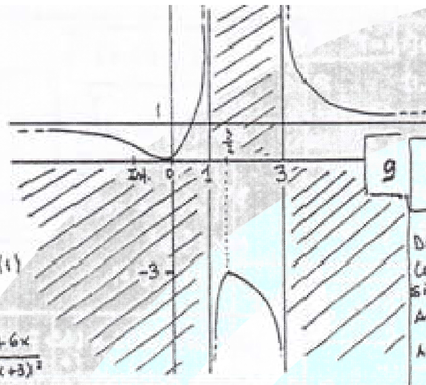
AV: $x=1$; $x=3$ AH: $y=1$

Creciente: $(0,1) \cup (1,3/2) = (0,\frac{3}{2}) - \{1\}$

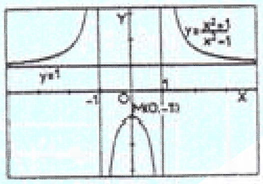
Máx: $(\frac{3}{2},-3)$ mín $(0,0)$

Inflexión:

$y' = \dfrac{-4x^2+6x}{(x^2-4x+3)^2}$



9 $\boxed{y = \dfrac{x^2+1}{x^2-1}}$



$D = \mathbb{R} - \{1,-1\}$

Corte: $(0,-1)$

Sim: par

AV: $x=1$; $x=-1$

AH: $y=1$; $y' = \dfrac{-4x}{(x^2-1)^2}$; Máx $(0-1)$; $y'' = \dfrac{12x^2+4}{(x^2-1)^3}$

$y''>0$ si $x \in (-\infty,-1) \cup (1,+\infty)$; $y''<0$ si $x \in (-1,1)$

2 $\boxed{y = \dfrac{x^2-4}{x+1}}$

$D = \mathbb{R} - \{-1\}$

Cortes: $(-2,0)$, $(2,0)$, $(0,-4)$

$y' = \dfrac{x^2+2x+4}{(x+1)^2}$

$y'>0$ $\forall x \in D$ creciente.

(num. no factorizable)

No extremos.

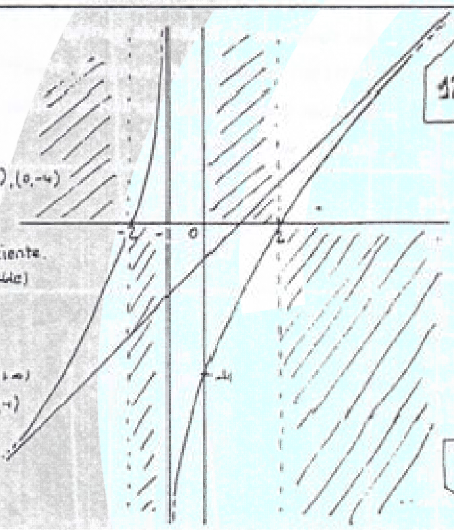
AV: $x=-1$

AO: $y=x-1$

Convexa $y''<0$; $(-1,+\infty)$

Cóncava $y''>0$ $(-\infty,-1)$

$y'' = \dfrac{-6}{(x+1)^3}$



12 $\boxed{y = \dfrac{x+1}{x-1}}$

$D = \mathbb{R} - \{1\}$

Corta: $(-1,0)$ $(0,-1)$

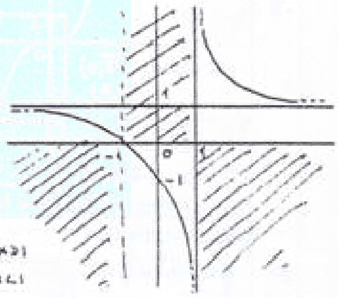
$y' = \dfrac{-2}{(x-1)^2} < 0$ $\forall x \in D$

decreciente.

$y'' = \dfrac{4}{(x-1)^3}$; $y''>0$ si $x>1$

$y''<0$ si $x<1$

AV: $x=1$; AH: $y=1$



13 $\boxed{y = \dfrac{x^2}{x-1}}$

$D = \mathbb{R} - \{1\}$

corta $(0,0)$

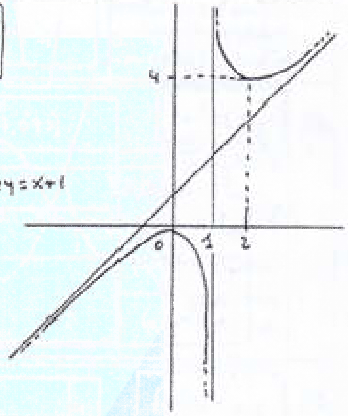
AV: $x=1$; AO: $y=x+1$

$y' = \dfrac{x(x-2)}{(x-1)^2}$

Máx $(0,0)$

mín $(2,4)$

$y'' = \dfrac{2}{(x-1)^3}$



3 $\boxed{y = \dfrac{x^3}{x^2-1}}$

$D = \mathbb{R} - \{1,-1\}$

Sim: impar ; corta: $(0,0)$

AV: $x=1$; $x=-1$; AO: $y=x$

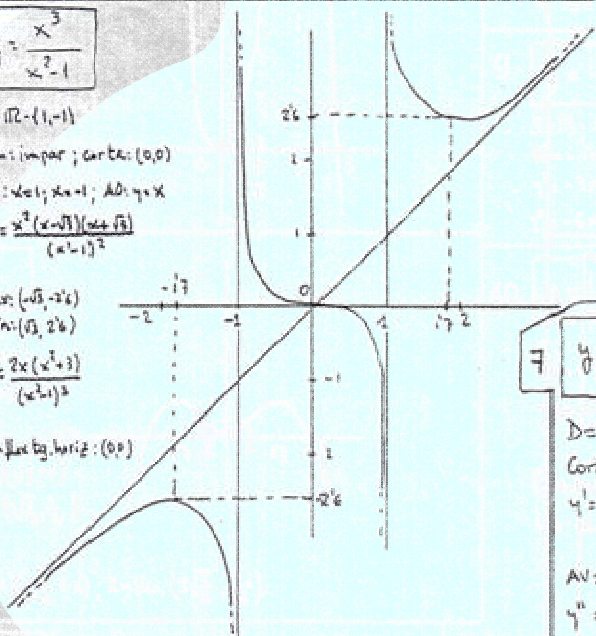
$y' = \dfrac{x^2(x-\sqrt{3})(x+\sqrt{3})}{(x^2-1)^2}$

Max: $(-\sqrt{3},-2'6)$

min: $(\sqrt{3},2'6)$

$y'' = \dfrac{2x(x^2+3)}{(x^2-1)^3}$

Inflex tg. horiz: $(0,0)$



7 $\boxed{y = \dfrac{x^2}{(x-1)^3}}$

$D = \mathbb{R} - \{1\}$

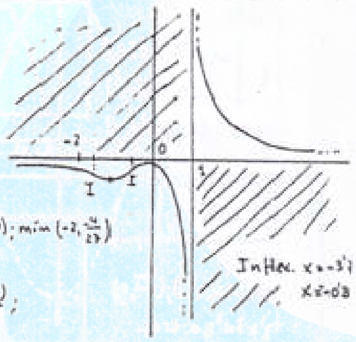
Corta $(0,0)$

$y' = \dfrac{-x(x+2)}{(x-1)^4}$;

Máx $(0,0)$; mín $(-2,\frac{-4}{27})$

AV: $x=1$; AH: $y=0$

$y'' = \dfrac{2(x+3'7)(x+0'3)}{(x-1)^5}$;



Inflex. $x=-3'7$

$x=-0'3$

4 $\boxed{y = \dfrac{x}{x^2+1}}$

$D = \mathbb{R}$

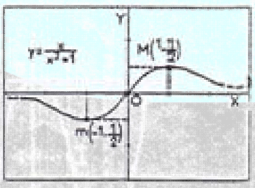
Corta: $(0,0)$

Sim: impar

AH: $y=0$

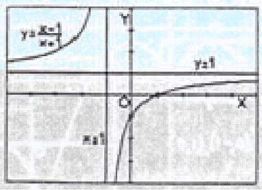
$y' = \dfrac{-(x^2-1)}{(x^2+1)^2}$

Máx $(1,1/2)$ mín $(-1,-1/2)$



$y'' = \dfrac{2x(x^2-3)}{(x^2+1)^3}$; Inflexiones: $(0,0)$ $(\sqrt{3},0'43)$ $(-\sqrt{3},-0'43)$

11 $\boxed{y = \dfrac{x-1}{x+1}}$



$D = \mathbb{R} - \{-1\}$

Corta: $(0,-1)$ $(1,0)$

AV: $x=-1$

AH: $y=1$

$y' = \dfrac{2}{(x+1)^2} \neq 0$ $\forall x \in D$, +: creciente.

8 $\boxed{y = \dfrac{x^2+11}{x+5}}$

$D = \mathbb{R} - \{-5\}$

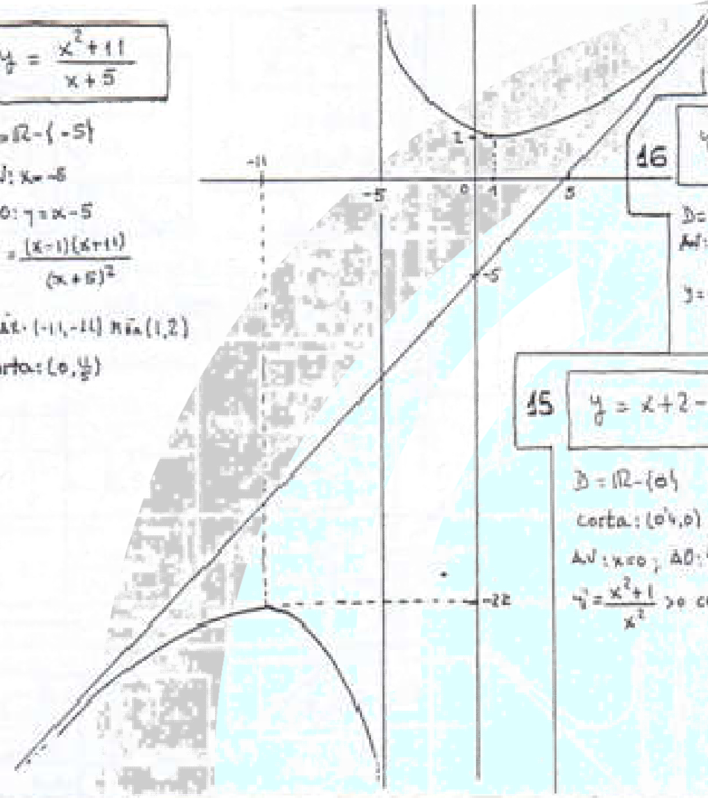
AV: $x = -5$

AO: $y = x - 5$

$y' = \dfrac{(x-1)(x+11)}{(x+5)^2}$

Máx. $(-11,-22)$ Mín $(1,2)$

Corta: $(0, \frac{11}{5})$



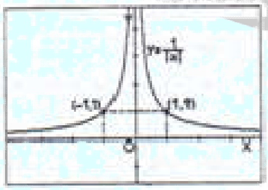
16 $\boxed{y = \dfrac{1}{|x|}}$

$D = \mathbb{R} - \{0\}$

AV: $x = 0$; AH: $y = 0$

$y = \begin{cases} \frac{1}{x} & x > 0 \\ -\frac{1}{x} & x < 0 \end{cases}$

Sim.: par



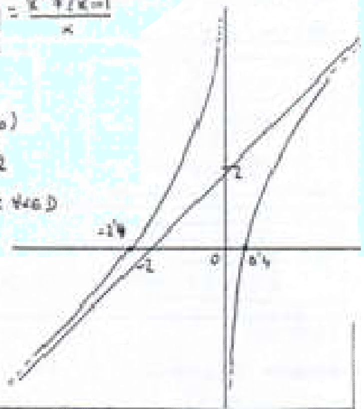
15 $\boxed{y = x + 2 - \dfrac{1}{x}} = \dfrac{x^2+2x-1}{x}$

$D = \mathbb{R} - \{0\}$

Corta: $(0'4,0)$, $(-2'4,0)$

AV: $x = 0$; AO: $y = x + 2$

$y' = \dfrac{x^2+1}{x^2} > 0$ creciente $\forall x \in D$



10 $\boxed{y = \dfrac{x^3}{(x-1)^2}}$

$D = \mathbb{R} - \{1\}$

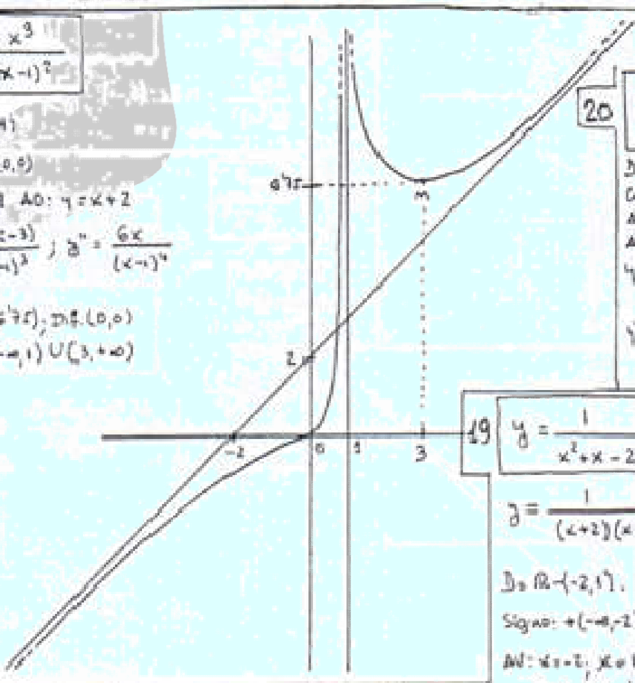
Corta: $(0,0)$

AV: $x = 1$ AO: $y = x + 2$

$y' = \dfrac{x^2(x-3)}{(x-1)^3}$; $y'' = \dfrac{6x}{(x-1)^4}$

mín $(3, 6'75)$; Inf. $(0,0)$

crece: $(-\infty,1) \cup (3,+\infty)$



20 $\boxed{y = \dfrac{x^2}{2-x}}$

$D = \mathbb{R} - \{2\}$

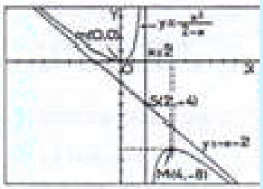
Corta: $(0,0)$

AV: $x = 2$;

AO: $y = -x - 2$

$y' = \dfrac{-x(x-4)}{(2-x)^2}$; Máx $(4,-8)$ mín $(0,0)$

$y'' = \dfrac{8}{(2-x)^3}$; no inflex. $y'' > 0$ si $x < 2$ $\cup$



19 $\boxed{y = \dfrac{1}{x^2+x-2}}$

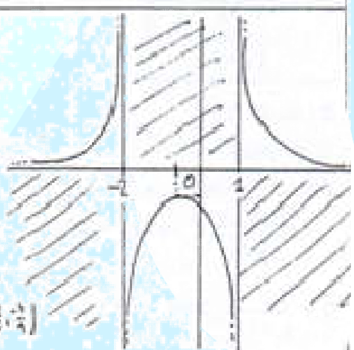
$y = \dfrac{1}{(x+2)(x-1)}$

$D = \mathbb{R} - \{-2, 1\}$

Signo: $+ (-\infty,-2) \cup (1,+\infty)$

AV: $x = -2$; $x = 1$; AH: $y = 0$

$y' = \dfrac{-2(x+1/2)}{(x^2+x-2)^2}$; Máx $\left(-\frac{1}{2}, -\frac{4}{9}\right)$



5 $\boxed{y = \dfrac{x^2+1}{x^2-3x+2}}$

$D = \mathbb{R} - \{2, 1\}$

Corta: $(0, 1/2)$

AV: $x = 1$; $x = 2$

AH: $y = 1$

$y' = \dfrac{-3(x+0'7)(x-1'4)}{(x-1)^2(x-2)^2}$

Máx: $x = 1'4$; mín: $x = -0'7$

Inflexión: I

17 $\boxed{y = \dfrac{1}{x^2+1}}$

$D = \mathbb{R}$

Signo: $+ \ \forall x$

Corta: $(0,1)$

AH: $y = 0$

$y' = \dfrac{-2x}{(x^2+1)^2}$ Máx $(0,1)$ $y'' = \dfrac{2(3x^2-1)}{(x^2+1)^3}$

14 $y = -x + 3 - \frac{4}{x}$

$D = \mathbb{R} - \{0\}$

$y = \frac{-x^2+3x-4}{x}$ = No factorizable

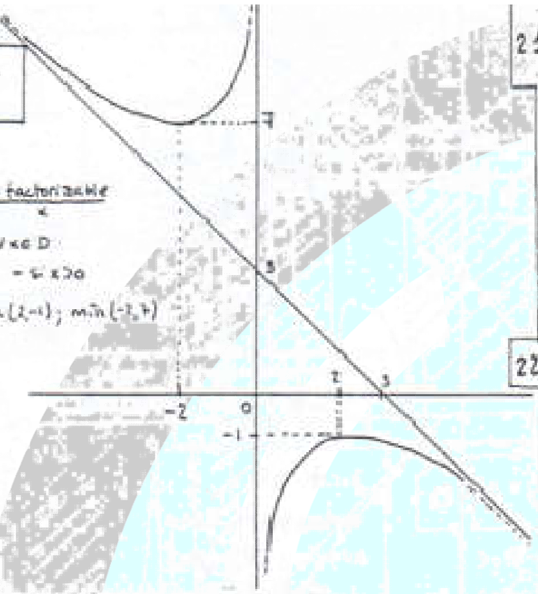
El numerador es $+ \forall x \in D$

Signo: + si $x<0$ − si $x>0$

$y' = \frac{-(x^2-4)}{x^2}$ Máx $(2,-1)$; mín $(-2,7)$

AV: $x=0$

AO: $y=-x+3$



21 $y = \frac{x^2}{x-2}$

$D = \mathbb{R} - \{2\}$

Corta: $(0,0)$

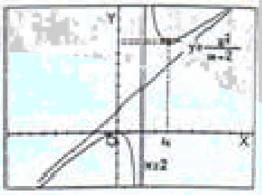
AV: $x=2$; AO: $y=x+2$

$y' = \frac{x(x-4)}{(x-2)^2}$;

Máx: $(0,0)$

mín: $(4,8)$

$y'' = \frac{8}{(x-2)^3}$; $y''>0$ $x>2$ ∪; $y''<0$ $x<2$ ∩; No infl.



22 $y = \frac{1}{x^3}$

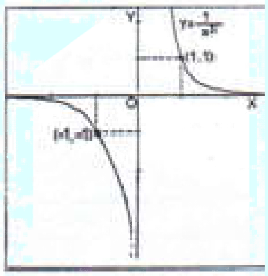
$D = \mathbb{R} - \{0\}$

Sim: impar

AV: $x=0$; AH: $y=0$

$y' = \frac{-3}{x^4}$ (<0 $\forall x \in D$)

decreciente.



18 $y = \frac{x^4+1}{x^3(x-1)}$

$D = \mathbb{R} - \{0,1\}$

No corta. Signo: + si $x>1$

AV: $x=1$, $x=0$; AO: $y=x+1$

$y' = \frac{x^5-2x^4-3x+2}{x^3(x-1)^2}$

Num. sin raíces enteras. Tiene 3 reales y 2 imagin. Dos máximos, un mínimo

La gráfica es parecida a esta. No sabemos las ordenadas de los extremos relativos.

25 $y = \frac{1}{x^2-1}$

$D = \mathbb{R} - \{1,-1\}$

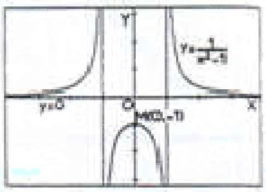
Corta: $(0,-1)$

AV: $x=1$; $x=-1$

Sim: par

$y' = \frac{-2x}{(x^2-1)^2}$; Máx: $(0,-1)$; $y'' = \frac{6x^2+2}{(x^2-1)^3}$; No infl.

AH: $y=0$



26 $y = \frac{x^3}{1-x^2}$

$D = \mathbb{R} - \{1,-1\}$

Corta: $(0,0)$

AV: $x=1$; $x=-1$

Sim: impar

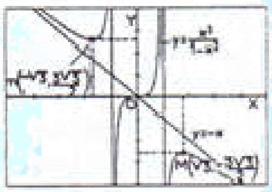
$y' = \frac{-x^2(x^2-3)}{(1-x^2)^2}$

Máx: $(\sqrt{3},-2'6)$

mín: $(-\sqrt{3},2'6)$

$y'' = \frac{2x(x^2+3)}{(1-x^2)^3}$; Inflex: $(0,0)$

AO: $y=-x$



23 $y = \frac{x^3}{(x+1)^2}$

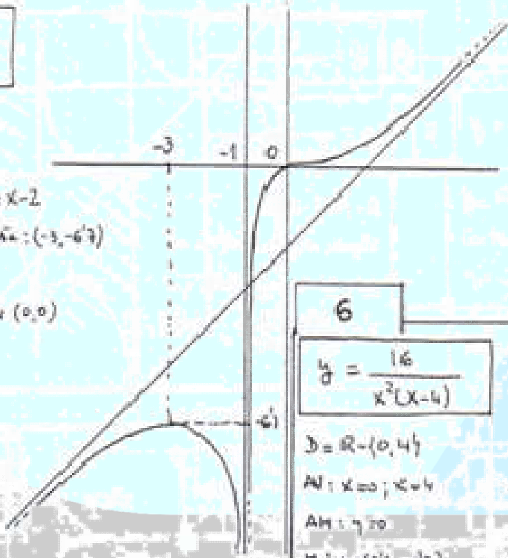
$D = \mathbb{R} - \{-1\}$

Corta $(0,0)$

AV: $x=-1$; AO: $y=x-2$

$y' = \frac{x^2(x+3)}{(x+1)^3}$; Máx: $(-3,-6'7)$

$y'' = \frac{6x}{(x+1)^4}$; Inflex $(0,0)$



6 $y = \frac{16}{x^2(x-4)}$

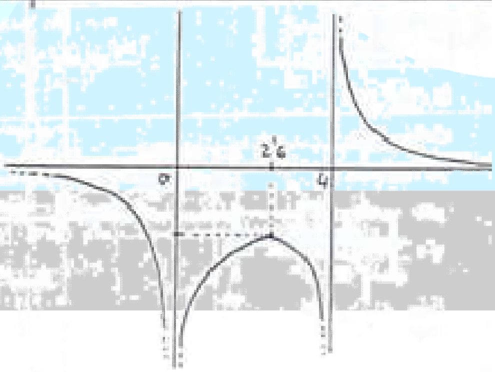
$D = \mathbb{R} - \{0,4\}$

AV: $x=0$; $x=4$

AH: $y=0$

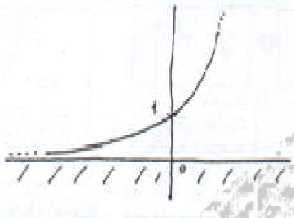
Máx: $(2'6,-1'7)$

$y' = \frac{-16(3x-8)}{x^3(x-4)^2}$



1. $\boxed{y = e^x}$

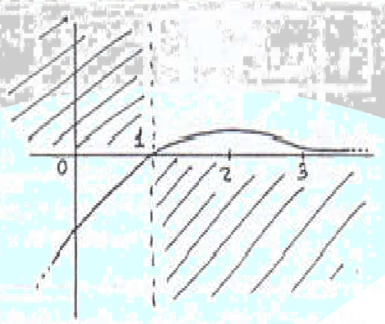
$D = \mathbb{R}$
Corta: $(0,1)$
AH: $y=0$
$y' = e^x > 0 \ \forall x$
$y'' = e^x > 0 \ \forall x$



HAY QUE SABÉRSELA

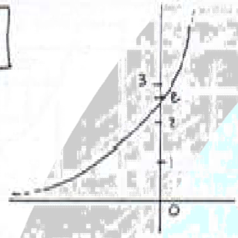
6. $\boxed{y = (x-1)e^{-x}}$

$D = \mathbb{R}$
Corta: $(0,-1)$ $(1,0)$
Signo: $+$ si $x>1$
AH: $y=0$
$y' = e^{-x}(2-x)$
Máx: $(2, e^{-2})$
$y'' = -e^{-x}(3-x)$
Inflex. $(3, 0'09)$



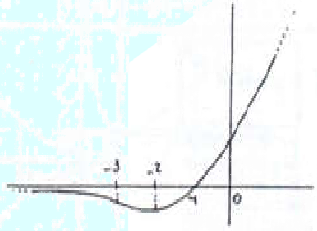
2. $\boxed{y = e^{x+1}}$

$D = \mathbb{R}$
Corta: $(0,e)$
$y>0 \ \forall x$
AH: $y=0$
creciente $\forall x$



7. $\boxed{y = (x+1)e^x}$

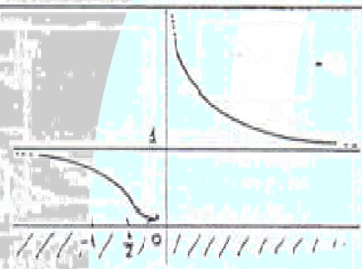
$D = \mathbb{R}$; corta: $(0,1)$ $(-1,0)$
Signo: $x>-1$ es $+$
$y' = (x+2)e^x$
crece: $x>-2$
Mín $(-2, -e^{-2})$
$y'' = (x+3)e^x$
Inflex: $(-3, -2e^{-3})$ } AH: $y=0$



3. $\boxed{y = e^{\frac{1}{x}}}$

$D = \mathbb{R} - \{0\}$
Signo: $+ \ \forall x \in D$
No corta
$y' = -\frac{1}{x^2} \cdot e^{1/x} < 0$
decreciente
$y'' = \frac{2x+1}{x^4} \cdot e^{1/x}$
$x = -\frac{1}{2}$ Inflex.
AV: $x=0$; AH: $y=1$; $\lim_{x\to 0^-} f(x) = 0$

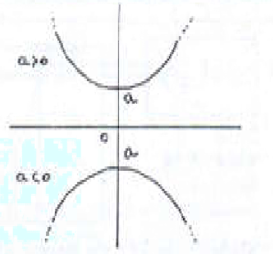
[Comparar con nº 4 Exp.]



8. $\boxed{y = a\,e^{x^2}}$

CASO $a>0$
$D = \mathbb{R}$, corta $(0,a)$
Sim. par; Signo: $+ \ \forall x$
$y' = 2ax\,e^{x^2}$; mín $(0,a)$
$y'' = 2ae^{x^2}[1+2x^2] > 0 \ \forall x$

CASO $a<0$: simétrica respecto eje x.

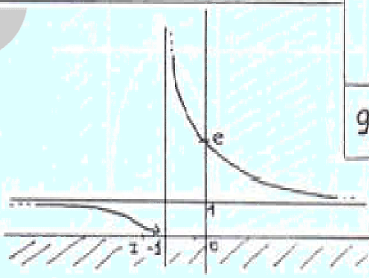


4. $\boxed{y = e^{\frac{1}{1+x}}}$

$D = \mathbb{R} - \{-1\}$
Signo: $+ \ \forall x \in D$
Corta: $(0,e)$
$y' = \frac{-1}{(1+x)^2} \cdot e^{\frac{1}{1+x}} < 0$
decrec. $\forall x \in D$
$y'' = \frac{(2x+3) \cdot e^{1/x+1}}{(x+1)^4}$
$x = -3/2$ Inflex.

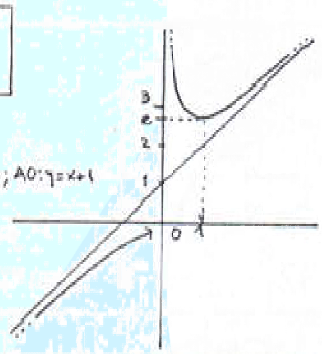
[Comparar con nº 3 Exp.]

AV: $x=-1$; AH: $y=1$; $\lim_{x\to -1^-} f(x) = 0$



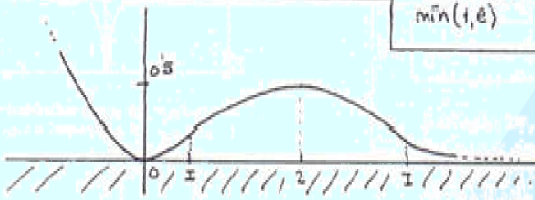
9. $\boxed{y = x \cdot e^{\frac{1}{x}}}$

$D = \mathbb{R} - \{0\}$
AV: $x=0$ dcha; AO: $y = x+1$
$\lim_{x \to 0^-} f(x) = 0$
$y' = e^{\frac{1}{x}}\left(\frac{x-1}{x}\right)$
mín $(1,e)$



5. $\boxed{y = x^2 e^{-x}}$

$D = \mathbb{R}$
Signo: $+ \ \forall x$
Corta: $(0,0)$
AH: $y=0$
$y' = x \cdot e^{-x}(2-x)$
mín $(0,0)$ Máx $(2, 0'5)$
$y'' = e^{-x}(x-3'4)(x-0'6)$; Infl. $x=3'4$; $x=0'6$

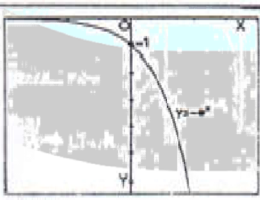


COMENTARIOS

- $a^{h(x)} > 0 \ \forall x \in D$
- El crecimiento exponencial es mucho más rápido que el de potencias
- Distinguir límite izda y dcha y $+\infty, -\infty$. – suelen tener asíntotas.
- $e \approx 2'7$

10. $\boxed{y = -e^x}$

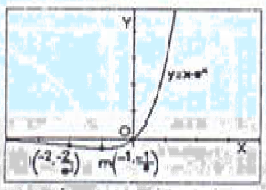
$D = \mathbb{R}$
Signo: $- \ \forall x$
Corte: $(0,-1)$
AH: $y=0$
$y' = -e^x < 0 \ \forall x$ decreciente



HAY QUE SABÉRSELA

11. $\boxed{y = x \cdot e^x}$

$D = \mathbb{R}$
Corta: $(0,0)$
Signo: $+$ si $x>0$
AH: $y=0$
$\lim_{x\to -\infty} xe^x = -\infty \cdot 0$; $\lim_{x\to -\infty} \frac{x}{e^{-x}} = \lim_{x\to -\infty} \frac{1}{-e^{-x}} = \frac{1}{-\infty} = 0$



$y' = e^x(x+1)$; $x>-1$ crece; mín $(-1, -0'37)$
$y'' = e^x(x+2)$; Inflex $(-2, -0'27)$

1 $\boxed{y = Lx}$

$D = (0, +\infty)$

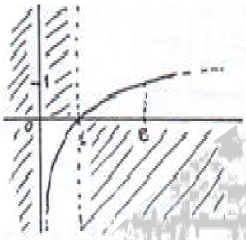
Signo: $Lx > 0$; $x > 1$ +

$Lx < 0$; $x < 1$ −

AV: $x = 0$

Corta: $Lx = 0$; $x = e^0 = 1$; $(1, 0)$

$y' = \frac{1}{x} > 0$ $\forall x \in D$ creciente



HAY QUE SABÉRSELA.

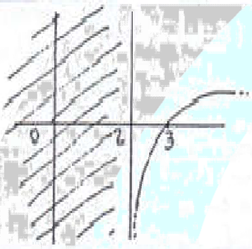
2 $\boxed{y = L(x-2)}$

$D = (2, +\infty)$

AV: $x = 2$

Corta $(3, 0)$

[Ver $y = Lx$; Nº 1 Log]



3 $\boxed{y = x \cdot Lx}$

$D = (0, +\infty)$

Corta: $(1, 0)$

Signo: $x > 1$ +

$y' = 1 + Lx$

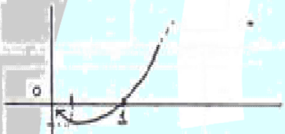
Crece: $Lx + 1 > 0$

$Lx > -1 \Rightarrow x > e^{-1}$

Mínimo: $1 + Lx = 0$; $Lx = -1$; $x = e^{-1}$ $\Rightarrow (e^{-1}, -e^{-1})$ mín.

$\lim_{x \to 0^+} x Lx = 0 \cdot (-\infty)$; $\lim_{x \to 0^+} \frac{Lx}{1/x} \cdots = 0$

$y'' = \frac{1}{x} > 0$ $\forall x \in D$



4 $\boxed{y = L(\operatorname{sen} x)}$

Periodo: 2π; $D_p: (0, \pi)$

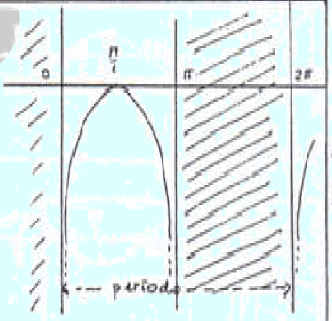
Cortes: $(\frac{\pi}{2}, 0)$

Signo: − $\forall x \in D$

Creciente: $(0, \frac{\pi}{2})$ Máx: $(\frac{\pi}{2}, 0)$

convexa $\forall x \in D$ $y'' < 0$

AV. $x = 0$; $x = \pi$



5. $\boxed{y = L|x|}$

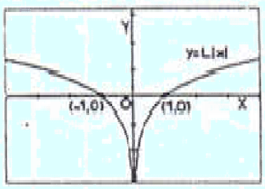
$D = \mathbb{R} - \{0\}$

Sim: par

Corta: $(1, 0)$ $(-1, 0)$

$y = \begin{cases} Lx & x > 0 \\ L(-x) & x < 0 \end{cases}$

AV: $x = 0$



La rama derecha es $y = Lx$

La rama izquierda es $y = -Lx$

6. $\boxed{y = \frac{1}{Lx}}$

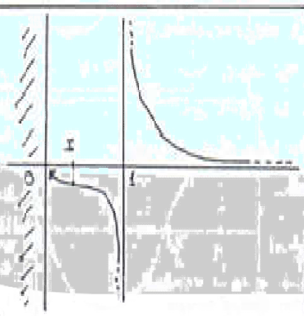
$D = (0, +\infty) - \{1\}$

AV: $x = 1$ AH: $y = 0$

$\lim_{x \to 0^+} \frac{1}{Lx} = 0$

$y' = \frac{-1}{x \cdot L^2 x} < 0$ decreciente

$y'' = \frac{Lx + 2}{x^2 \cdot L^3 x}$; $(e^{-2}, -\frac{1}{2})$ Inflx.



7 $\boxed{y = L(x^2 - 4)}$

$D = \mathbb{R} - [-2, 2]$

Sim: par

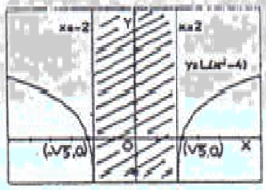
Corte: $(\pm\sqrt{5}, 0)$

Signo: $L(x^2-4) > 0$

$x^2 - 4 > 1$; $x^2 - 5 > 0$; + $(-\infty, -\sqrt{5}) \cup (\sqrt{5}, +\infty)$

$y' = \frac{2x}{x^2 - 4}$; $x > 0$ $(x > 2)$ crece.

AV: $x = 2$; $x = -2$



8. $\boxed{y = x - L(1+x)}$

$D = (-1, +\infty)$

Corta $(0, 0)$

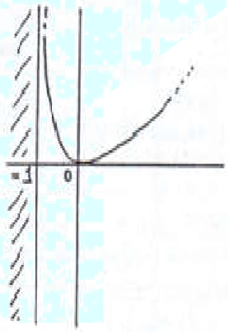
Signo: + $\forall x \in D$

AV: $x = -1$

$y' = \frac{x}{x+1}$ Crece: $x > 0$

mín: $(0, 0)$

$y'' = \frac{1}{(1+x)^2} > 0$ $\forall x \in D$

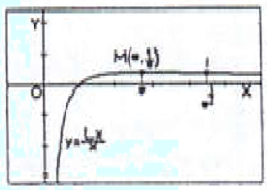


9. $\boxed{y = \frac{Lx}{x}}$

$D = (0, +\infty)$

Corta: $(1, 0)$

Signo: + $x > 1$



$y' = \frac{1 - Lx}{x^2}$

$y' = 0$ si $Lx = 1 \Rightarrow x = e$; máx $(e, e^{-1}) = (e, 0'36)$

$y'' = \frac{2Lx - 3}{x^3}$; $y'' = 0$ si $Lx = \frac{3}{2}$; $x = \sqrt{e^3} = 4'48$; Inflex: $(4'48, 0'33)$

AH: $y = 0$; $\lim_{x \to +\infty} \frac{Lx}{x} = \lim_{x \to +\infty} \frac{1/x}{1} = 0$

10 $\boxed{y = L(x-1)}$

$D = (1, +\infty)$

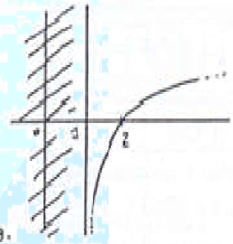
AV: $x = 1$

Corta $(2, 0)$

$y' = \frac{1}{x-1} > 0$ $\forall x \in D$ crece

$y'' = \frac{-2}{(x-1)^2} < 0$ $\forall x \in D$ ∩

Comparar con Nº 85 (-2 Log.



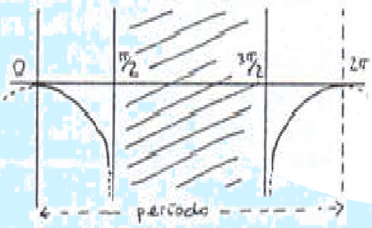
11. $\boxed{y = L(\cos x)}$

Periodo: 2π.

$D_p: [0, 2\pi] - [\frac{\pi}{2}, \frac{3\pi}{2}]$

AV: $x = \frac{\pi}{2}$; $x = \frac{3\pi}{2}$

Máx: $(0, 0)$, $(2\pi, 0)$



12. $\boxed{y = Lx^2}$

$D = \mathbb{R} - \{0\}$

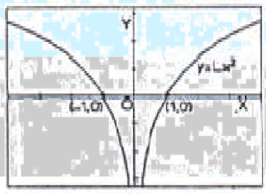
Corte: $(1, 0)$ $(-1, 0)$

Sim: par

Signo: $Lx^2 > 0$

$x^2 > 1$; $(x-1)(x+1) > 0$

+ si $x \in (-\infty, -1) \cup (1, +\infty)$ $\Big\}$ $y' = \frac{2}{x}$; $y' > 0$ si $x > 0$ crece.



11.